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A Fuzzy Approach for Bi-Level Programming Problems

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ABSTRACT. Bi-level programming problem (BLPP) could affect the final decision; its result is a mutually coordinated scheme among all layers and can help solve complex practical problems. Methods to solve the BLPP have been summarized and described in this paper. A fuzzy method was applied to solve the BLPP by using the concept of coupled membership function for multi-objective optimization programming. In addition, a numerical example has been employed to show the calculation processes. It is also proposed that the future research direction is the solution of fuzzy optimization problems in multi-level programming problems.

Keywords: bi-level programming, Kth-best algorithm, fuzzy approach

1. Introduction

The previous bilevel programming (BP) method with two-level structure was developed to reflect the trade-off between two decision levels with different objectives (Camacho, 2015). When solving BP programming, the leader-follower decision strategy is introduced into the algorithm optimization process, so as to achieve the optimal solution that is satisfactory to two levels Decision Makers (DMs). Different decision management systems usually have their own starting pointswhen dealing with some system problems, so sometimes the ideal solutions will conflict. In previous studies, it can be learned that in addition to effectively balancing DMs requirements at different levels, the limitations of BP algorithm will be reflected in setting system parameters. Therefore, the research direction can be aimed at solving the uncertainty in the system by combining bi-level programming, fuzzy numbers and interval numbers, and using accurate and appropriate algorithms. The fuzzy and interval numbers are transformed into explicit values, so as to acquire an effective optimization scheme (Ma et al., 2017).

Scholars have proposed many methods to solve multi-level programming problem (MLPP). The majority of them are concepttual approaches to vertex enumeration and transformation. The vertex enumerationr is to find the control variable with a higher level of compromise point using the adjusted simplex algorithm. For some large planning problems, it is less efficient and will lose generality. The latter involves modifying the programming problem condition or penalty function of the upper-level constraint constructed by Kuhn-Tucker (K-T) at the lower level. Because nonlinearities or Lagrangian terms appear in the constraints, the auxiliary problem becomes complex and sometimes unmanageable.

Aiming at the problem of low computational efficiency of the existing methods, this paper proposes a fuzzy method to solve the above problems by using the membership functions are coupled into multi-objective optimization.Solution search depends on membership function changes that represent the satisfaction of possible solutions to the decision of two levels, rather than vertex enumeration, and does not generate higher-order constraints.We do not assume that optimal solutions exist at corner points, in contrast to vertex enumeration. Due to the difficulty of defining a reliable optimality in multi-person decision making processes, and limiting potential solutions to corner points is by definition a problem, we believe that the notion of satisfaction is more acceptable than optimality. In the nondominated region, there may be potential solutions that are satisfactory. Therefore, it is very efficient, and increases the original problem's complexity in no way (Bard, 1983), Wen and Hsu's bicriteria algorithm (Wen and Hsu, 1989) and the two-stage method (Wen and Hsu, 1992) are used to solve the auxiliary MLPP.

Hence, this paper discusses how to solve bi-level programming problems (BLPP) using fuzzy methods. The article first describes the method used before. The Kth-best approach is discussed in part 2 and the fuzzy concepts and methods are applied to solve the BLPP. An example of the proposed method is provided in the paper in order to demonstrate it, and compares it with the classical solution. The final section concludes with comments and future research directions.

2. Methodology

2.1. The Traditional Method

The idea of bi-level programming is a mathematical model of optimization in which problems at different levels have their own objective functions and constraints, and there are two levels

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of hierarchical structure. Upper-level decision variables determine the objective function and constraints in upper-level planning, but they are also constrained by the lower-level planning's optimal solution, which is affected and constrained by the upper-level decision variables. It can simultaneously consider the interests of both the global and the individual, ensuring that the global is first and the local is second. Then the final plan to solve complex practical problems is often the result of the mutual coordination of the two levels of goal planning (Lv, 2009).

To solve these constrained multi-level programming problems, Basar and Olsder (1982) proposed the Kth-best algorithm: Method First, we search for the individual optimal solution in the upper DM. As long as the solution is compatible with the optimal solution found in the lower DM, we obtain an optimal solution. Searching continues until the upper DM and lower DM optimal solutions match each other if there is no match between the two corners (extreme values) of the previous point. In the process of solving this algorithm, we can see an implicit compromise process. The DM in the upper layer reduces its own objective value to make a compromise for the optimal solution of the DM in the lower layer. As a last example, we consider an extreme case in which two independent optimal solutions of two DMS are located at two adjacent vertices. When the Kth-best algorithm is applied, it will force its solution to be one of them, and then the DM acting first will dominate the optimal solution. (Jenkins and Passino, 1999). Since the conflict is not yet resolved, the Kth-best solution seems to make little sense. It is more urgent to find a solution to these two extremes. This conflict phenomenon is reflected in many problems. The basic idea of the Kth-best algorithm is that the optimal points must exist between the corners, and the search of the corners leads to a complex enumeration process. There is difficulty defining a reliable optimality for multi-person decision problems. Even in non-cooperative phenomena, compromise or coordination is usually required in order to reach a solution.

Kth-best algorithms have a slow convergence rate when they are applied to large-scale problems, as they have to search a large number of vertices at once. DMs are unaware of the levels of relationships and the possible effects of individual actions, that is, there is a lack of clear information especially about the objectives for each DM's achievement. Secondly, the program shows that the rate of return always occurs in the upper DM, while lowering the DM lets some profit when exploiting the previous loss, that is, decreasing the order from above is best if the low level is not satisfactory. Even in a decentralized organization, non-dominant solutions may be more appropriate than classical ones. A non-angular, non-dominated solution may be better in this case, since it avoids the computational difficulties caused by enumeration. The Kth-Best solution is dominant in many other cases and thus is not very attractive to any DM, particularly in practice.

2.2. Fuzzy Approach for BLPP

Shih (1996) proposed an algorithm for supervised search (supervised by the top-level DM) that could help produce (non-dominated) satisfactory solutions to the multilevel planning problem. This paper applies the method of the literature to solve the multi-level programming, and lists the fuzzy algorithms of bi-level programming separately. The upper DMs specify preferences with some leeway for control variables and objectives in this solution process. Fuzzy set theory models this information as a membership function and passes it to the subordinate DM as an additional constraint. The goals of lower-level DMs need to be optimized not only for themselves, but also satisfy the goals and preferences of higher-level decision makers as much as possible. Without careful consideration of the goals and preferences at the top level, the proposed solution is likely to be rejected and the search for a solution will be a lengthy process. If the upper DM agrees with the lower DM's proposed solution, then the lower DM has reached a satisfactory solution. This proposal will need to be reevaluated and changed if rejected, as well as any corresponding margins or tolerances, until a satisfactory outcome can be reached. This strategy does not violate the noncooperative property, where the two-level decision system first seeks its own optimal solution in isolation. It does, however, necessitate some collaboration with traditional methods.

The DM at the higher level first resolves the following issue mathematically:

$$\text{Max } f_1(x_1, x_2) = c_{11}x_1 + c_{12}x_2 \tag{1a}$$

$$s.t. \quad (x_1, x_2) \in F_2 = \left\{(x_1, x_2) \middle| A_1x_1 + A_2x_2 \le b_1x_1 \; and \; x_2 \ge 0\right\} \tag{1b}$$

(x_1^U, x_2^U, f_1^U) is assumed to be the solution, and the lower-level DM solves these problems independently:

$$\text{Max } f_2(x_1, x_2) = c_{21}x_1 + c_{22}x_2 \tag{2a}$$

$$s.t. \quad (x_1, x_2) \in F_2 = \left\{(x_1, x_2) \middle| A_1x_1 + A_2x_2 \le b_1x_1 \; and \; x_2 \ge 0\right\} \tag{2b}$$

Solution is assumed to be (x_1^L, x_2^L, f_2^L). The above solutions are then disclosed to both DMs. If (x_1^U, x_2^U) = (x_1^L, x_2^L), we arrive at an optimal solution. In general, two solutions differ due to conflicts in nature between the two objective perspectives. It is obvious that the upper-level DM cannot use the optimal decision x_1^U to control the lower-level DM. It makes more sense to provide for some flexibility or tolerances that give lower-level DMs a larger viable domain to find their ideal answer and that significantly cut down on the amount of time or iterations required to accomplish so. On x_1, the decision range should be "around x_1^U with its maximum tolerances P_1". x_1^U is the most preferred decision; a decision at $x_1^U - P_1$ and $x_1^U + P_1$ is the worst possible one; a linear increase in satisfaction or preference within [$x_1^U - P_1$, x_1^U] and a linear decrease within [x_1^U + P_1, x_1^U] can be observed; other decisions are not acceptable. In fuzzy set theory, these membership functions can be formulated as follows (Zhang et al., 2019):

$$\mu_{x_1}(x_1) = \begin{Bmatrix} \left[x_1 - (x_1^U - P_1)\right] / P_1, & if \; x_1^U - P_1 \le x_1 \le x_1^U \\ \left[(x_1^U + P_1) - x_1\right] / P_1, & if \; x_1^U - x_1 \le x_1^U + P_1 \\ 0, & otherwise \end{Bmatrix} \tag{3}$$

It is also depicted in Figure 1. At the same time, it is important for the upper DM to be tolerant enough to the lower DM to be clear about its goals in order to guide or supervise the lower DM in the right direction to find a solution. In general, an upper-level DM can legitimately take into account that preference inside [$f_1{'}$, f_1^U] is lin- early growing, all $f_1 \geq f_1^U$ are absolutely acceptable, and all $f_1 < f_1{'}$ [$=f_1(x_1^L, x_2^L)$] are completely unacceptable. Since the lower-level DM obtained the optimal value at (x_1^L, x_2^L) and then gave the upper-level DM the objective value of $f_1{'}$, any $f_1 < f_1{'}$ is unattractive. In this case, it is reasonable to assume the fol- lowing member function:

$$\mu_{f_1}[f_1(x_1)] = \begin{Bmatrix} 1, & if\ f_1(x) > f_1^U \\ \dfrac{f_1(x) - f_1{'}}{f_1^U - f_1{'}}, & if\ f_1{'} \leq f_1(x) \leq f_1^U \\ 0, & if\ f_1(x) < f_1 \end{Bmatrix} \tag{4}$$

Figure 1 illustrates this as well. In the lower-level DM, the objective is optimized under "x1 is about x1U" constraints and "There is some way in which f1 is nearer or greater than f1U" which are modeled by the membership functions (3) and (4). As a result of model (5) or (6), the lower-level DM obtains the following problem (Lai and Hwang, 1993):

$$\underset{x_2}{\text{Max}}\ f_2(x_1, x_2) = c_{21}x_1 + c_{22}x_2 \tag{5a}$$

subject to:

$$\begin{aligned} & A_1x_1 + A_2x_2 \leq b, \\ & x_1 = x_1^U, \\ & f_1(x) \geq f_1^U, \\ & x_1\ and\ x_2 \geq 0 \end{aligned} \tag{5b}$$

$$\underset{x_2}{\text{Max}}\ f_2(x_1, x_2) = c_{21}x_1 + c_{22}x_2 \tag{6a}$$

subject to:

$$\begin{aligned} & A_1x_1 + A_2x_2 \leq b, \\ & \mu_{x_1}(x_1) \geq \alpha I, \\ & \mu_{f_1}[f_1(x)] \geq \beta, \\ & x_1\ and\ x_2 \geq 0, \\ & \alpha \in [0,1]\ and\ \beta \in [0,1] \end{aligned} \tag{6b}$$

Accordingly, α (a row vector) represents the minimum acceptable degree of satisfaction or preference for the decision x_1, and β represents the minimum acceptable degree of preference for the objective; I has the same size as $\mu_{x1}(x_1)$ or x_1 and is a column vector whose members are all equal to 1. The feasible ranges constrained by $\mu_{x1}(x_1) \geq \alpha$ and $\mu_{f1}[f_1(x)] \geq \beta$ are depicted in Figure 1. Lower-level DMs are obviously capable of analyzing various solutions that satisfy upper-level DMs' satisfactory levels α and β.

For each possible solution available to the upper-level DMs, the lower-level DMs may be willing to build a membership function for the objective so that they can rate the satisfaction of each potential solution. Here, assume that the lower-level DMs have the following membership function for the goal:

$$\mu_{f_2}[f_2(x)] = \begin{Bmatrix} 1, & if\ f_2(x) > f_2^U \\ \dfrac{f_2(x) - f_2{'}}{f_2^U = f_2{'}}, & if\ f_2{'} \leq f_2(x) \leq f_2^U \\ 0, & if\ f_2(x) < f_2{'} \end{Bmatrix} \tag{7}$$

where $f_2{'} = f_2(x_1^T)$. As can be seen, μ represents a one-to-one mapping within the compact intervalf_2^L and $f_2{'}$. Because f_2^L is the best solution of (6), $f_2(x_1) > f_2^L$ is impossible, while the upper-level DM gives more constraints to the lower-level DM. The lower-level DM will not accept any $f_2(x) > f_2{'}$ for the same reason as the upper-level DM, discussed above. Therefore, the lower-level DM has $\mu_{f2}[f_2(x)] = [f_2(x) - f_2{'}] / [f_2^U - f_2{'}]$ and the following auxiliary model (8) or (9):

$$\text{Max}\ \delta = \mu_{f_2}[f_2(x)] \tag{8a}$$

subject to:

$$\begin{aligned} & A_1x_1 + A_2x_2 \leq b, \\ & \mu_{x_1}(x_1) \geq \alpha I, \\ & \mu_{f_1}[f_1(x)] \geq \beta, \\ & x_1\ and\ x_2 \geq 0, \\ & \alpha \in [0,1]\ and\ \beta \in [0,1] \end{aligned} \tag{8b}$$

$$\text{Max}\ \delta \tag{9a}$$

subject to:

$$\begin{aligned} & A_1x_1 + A_2x_2 \leq b, \\ & \mu_{x_1}(x_1) \geq \alpha I, \\ & \mu_{f_1}[f_1(x)] \geq \beta, \\ & \mu_{f_2}[f_2(x)] \geq \delta, \\ & x_1\ and\ x_2 \geq 0, \\ & \alpha \in [0,1]\ and \beta \in [0,1] \end{aligned} \tag{9b}$$

where δ is the satisfactory degree of the lower-level DM who searches for a solution with a higher δ value under the consideration of α and β values.

To resolve conflict between both DMs and to avoid the upper-level DM's rejection, the lower-level DM should try to maximize α, β, and δ simultaneously, that is:

$$\text{Max}\{\delta, \alpha, \beta\} \tag{10a}$$

subject to:

$$\begin{aligned}
&A_1x_1 + A_2x_2 \le b, \\
&\mu_{x_1}(x_1) \ge \alpha I, \\
&\mu_{f_1}[f_1(x)] \ge \beta, \\
&\mu_{f2}[f_2(x)] \ge \delta, \\
&x_1 \ and \ x_2 \ge 0, \\
&\alpha \in [0,1] \ and \ \beta \in [0,1]
\end{aligned} \tag{10b}$$

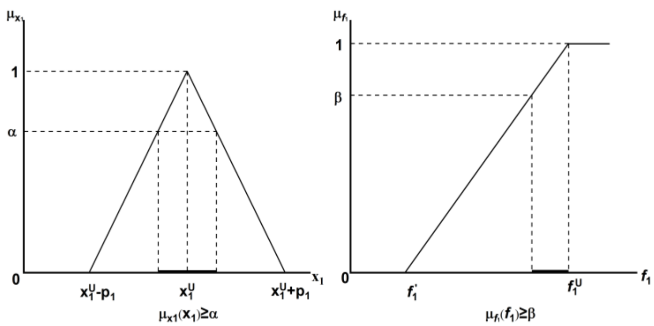

Figure 1. The membership functions for x_1 and f_1.

If the min operator is used to aggregate the satisfactory levels or $\lambda = \min\{\alpha, \beta, \delta\}$, the above problem will be expressed by model (11) or (12):

$$\text{Max}\ \lambda \tag{11a}$$

subject to:

$$\begin{aligned}
&A_1x_1 + A_2x_2 \ge b, \\
&\mu_{x_1}(x_1) \ge \lambda I, \\
&\mu_{f_1}[f_1(x)] \ge \lambda, \\
&\mu_{f_2}[f_2(x)] \ge \lambda, \\
&x_1 \ and \ x_2 \ge 0, \\
&\lambda \in [0,1]
\end{aligned} \tag{11b}$$

$$\text{Max}\ \lambda \tag{12a}$$

subject to:

$$\begin{aligned}
&A_1x_1 + A_2x_2 \le b, \\
&A_1x_1 + A_2x_2 \le b, \\
&[(x_1^U + p_1) - x_1] / p_1 \ge \lambda I, \\
&[x_1 - (x_1^U - p_1)] / p_1 \ge \lambda I, \\
&\mu_{f_1}[f_1(x)] = [f_1(x) - f_1'] / [f_1^T - f_1'] \ge \lambda, \\
&\mu_{f_2}[f_2(x)] = [f_2(x) - f_2'] / [f_2^L - f_2'] \ge \lambda, \\
&x_1 \ and \ x_2 \ge 0, \\
&\lambda \in [0,1]
\end{aligned} \tag{12b}$$

According to the application of Bellman and max-min decisions, there is a fuzzy max-min programming problem in model (9) (Ren and Zhang, 2018).

As long as model (9) gives a satisfactory solution to the upper DM's, the scheme is solved. In other cases, they should provide a new membership function that adds constraints to the objectives and control variables until a mutually satisfactory outcome is reached. The solution obtained by combining the control decision set and the objective with tolerance is a satisfactory solution.

The linear (and triangular) form is chosen for its computational efficiency since many nonlinear functions can be transformed into equivalent linear forms by variable transformation. So it is without loss of generality to discuss only the linear form in this paper. In fact, membership functions are indispensable when applying fuzzy methods in problem solving. In order to determine a membership function, heuristics are applied, reliability concerns are taken into account, and theoretical requirements are considered. In this study, we will not discuss various methods and functional forms for generating membership functions. Lai and Hwang have provided a concise discussion of these related topics (Lai and Hwang, 1993).

3. The Numerical Example

We demonstrate the proposed method with the following example. An export-oriented country concentrates on the production of two important products 1 and 2, which are manufactured by firms with some capacity. The profit is \$1 per unit for product 1 and \$2 per unit for product 2. Product 1 can be exported, earning \$2 per unit from abroad, while product 2 needs to import raw materials at \$1 per unit. There are two levels of DMs associated with this situation, the government (superior) and the company manager (subordinate), each of whom can only deal with one decision variable, x_1 and x_2, respectively. There are two objectives: (i) maximum exports, $f_1(x)$, and (ii) maximum profit on the product, $f_2(x)$. Therefore, we can formulate the problem as follows:

$$\underset{x_1}{\text{Max}}\ f_1 = 2x_1 - x_2 \ \ (\textit{effect on the export trade}) \tag{13}$$

where x_2 solves:

$$\underset{x_2}{\text{Max}}\ f_2 = x_1 + 2x_2 \ \ (\textit{profit on the products}) \tag{14a}$$

subject to:

$$\begin{aligned}
&3x_1 - 5x_2 \le 15 \ \ (\textit{capacity}), \\
&3x_1 - x_2 \le 21 \ \ (\textit{management}), \\
&3x_1 + x_2 \ge 27 \ \ (\textit{space}), \\
&3x_1 + 4x_2 \le 45 \ (\textit{material}), \\
&x_1 + 3x_2 \le 30 \ \ (\textit{labor hours}), \\
&x_1 \ and \ x_2 \ge 0
\end{aligned} \tag{14b}$$

where constraint set is denoted by X. The Kth-best solution is $(x_1, x_2) = (8, 3)$ at $K = 2$. In addition, the optimum for the upper-level objective is $f_1 = 13.5$ at (7.5, 1.5) and for the lower-level objective is $f_2 = 21$ at (3, 9). The decision variable and objective function spaces are shown in Figure 2.

The proposed approach first finds individual optimal solutions by solving Equations (5) and (6) to obtain $(x_1^U, x_2^U) = (7.5, 1.5)$ and $f_1^U = 13.5$ as well as $x_1^L, x_2^L = (3, 9)$ and $f_1^L = 21$. $f_1^T = 13.5$ and let us assume $f_1' = 0$ (only positive is meaningful here) instead of –3 and $f_2' = 10.5$. Take the upper-level DM's control decision x_1 to be around 7.5, with positive and negative side tolerances of 4.5 and 0.5, respectively. By Equations (3), (4) and (6), membership functions $\mu_{x1}(.)$, $\mu_{f1}(.)$ and $\mu_{f2}(.)$ are built. The lower-level DM then solves the following problem of Equation (15):

$$\text{Max}\,\lambda \tag{15a}$$

subject to:

$$\begin{aligned} & x \in X, \\ & x_1 \geq 4.5\lambda + 3, \\ & x_1 \leq 8 - 0.5\lambda, \\ & 2x_1 - x_2 \geq 13.5\lambda, \\ & x_1 + 2x_2 - 10.5 \geq 10.5\lambda, \\ & \lambda \in [0,1] \end{aligned} \tag{15b}$$

whose compromise solution is $f^* = (f_1^*, f_2^*) = (9.29, 17.72)$ at $x^* = (7.26, 5.23)$ with the overall satisfaction of both DMs $\lambda = 0.69$. Realized satisfactory levels are $(\mu_{x1}^*, \mu_{f1}^*, \mu_{f2}^*) = (0.95, 0.69, 0.69)$. If the upper-level DM's total satisfactory level $\lambda_1 = \min\{\mu_{x1}^*, \mu_{f1}^*\}$, then our solution provides $\lambda_1 = 0.69$ and λ_2 (of the lower-level DM) = 0.69. On the other hand, the Kth-best solution $f = (13, 14)$ at $x = (8, 3)$ has $(\mu_{x1}, \mu_{f1}, \mu_{f2}) = (0.0, 0.96, 0.33)$ and thus $\lambda_1 = 0.0$ and $\lambda_2 = 0.33$. As far as satisfactions of both DMs are concerned, our solution is clearly better than the Kth-best.

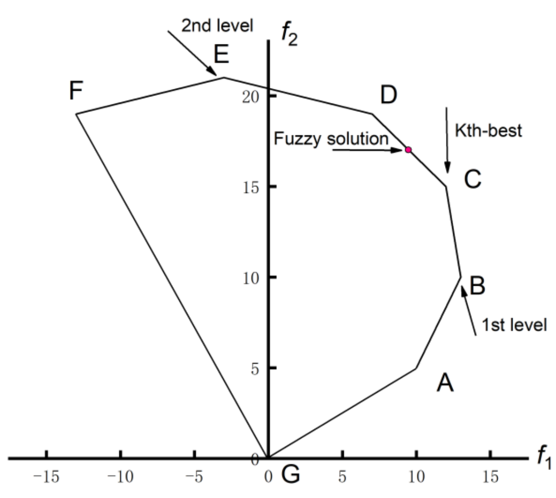

Figure 2. The objective function space for the example.

4. Conclusions

This paper summarizes and describes previously used bi-level programming methods, applies a fuzzy approach to solving bi-level programming problems, and a numerical example is given to solve it, and concludes by offering recommendations for future research. The method applied in this paper to solve BLPP, in the search process does not rely on the enumeration of vertices. Instead, the model is solved by the change of membership function. So even large-scale problems in life can be scaled with simple calculations. For nonlinear programming problems, this method has no great improvement, but at least it does not increase the order and make the problem more complicated.

The above member functions and method calculations provide satisfactory solutions, so the next research direction is mainly to explore a variety of functions and operators, so that different DMs can change the function form in the interaction algorithm discussed above to find the optimal solution; MLPP can therefore be solved with a complete decision support system. Multilevel programming problems involving nonlinear functions, integers, or mixed integers (nonlinear functions) can also be solved with this method. In multi-level programming problems, there is often a lack of accuracy in the parameters or input data. Consequently, a useful and intriguing direction for future research is the development of techniques for handling fuzzy problems and novel concepts for tackling multilayer planning problems.

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